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MATEMÁTICA APLICADA I

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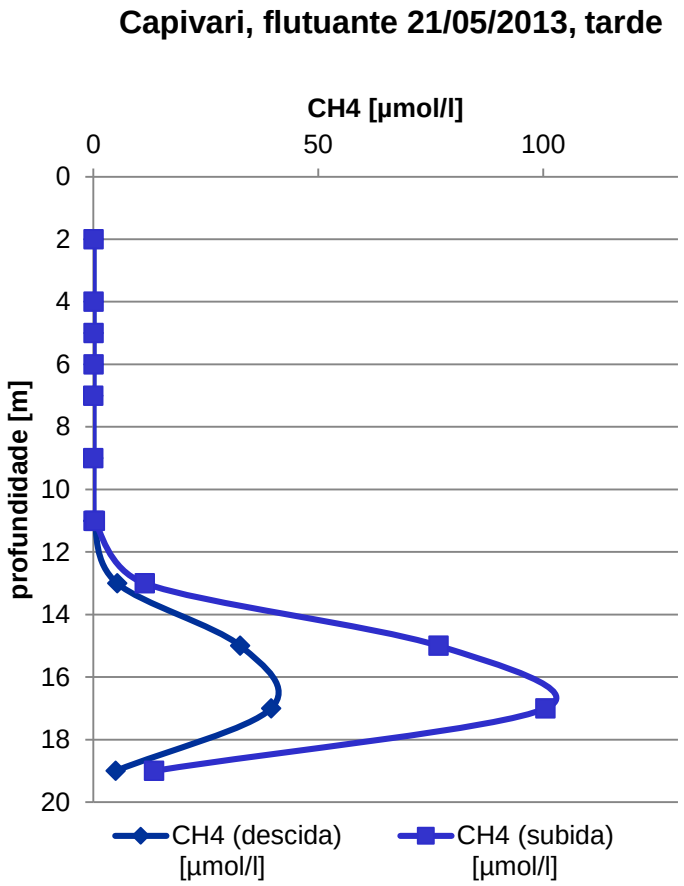
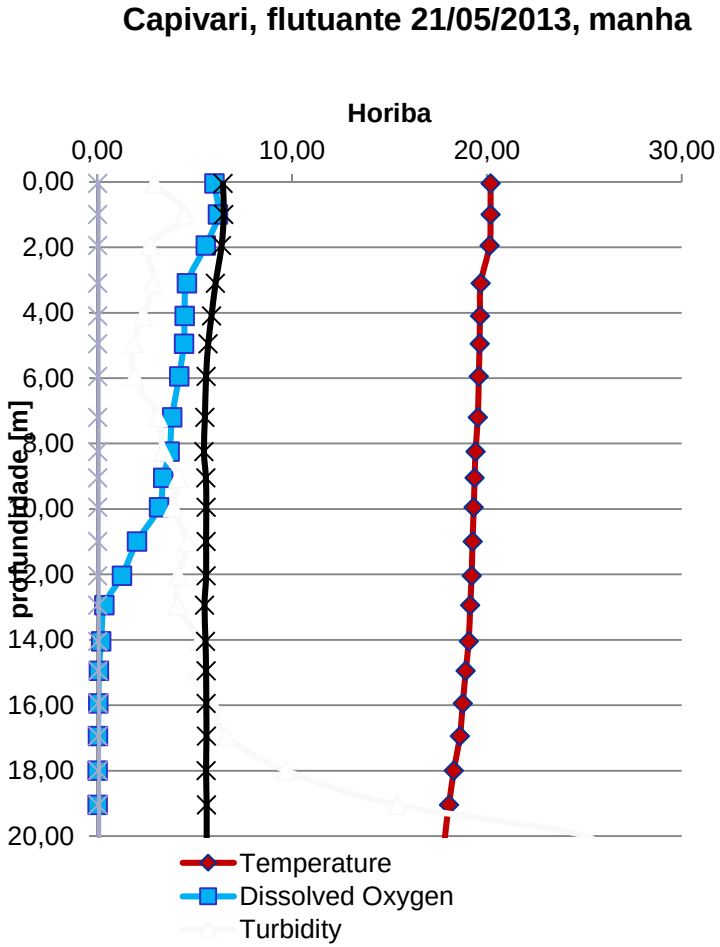
CH4 e T, DO, …





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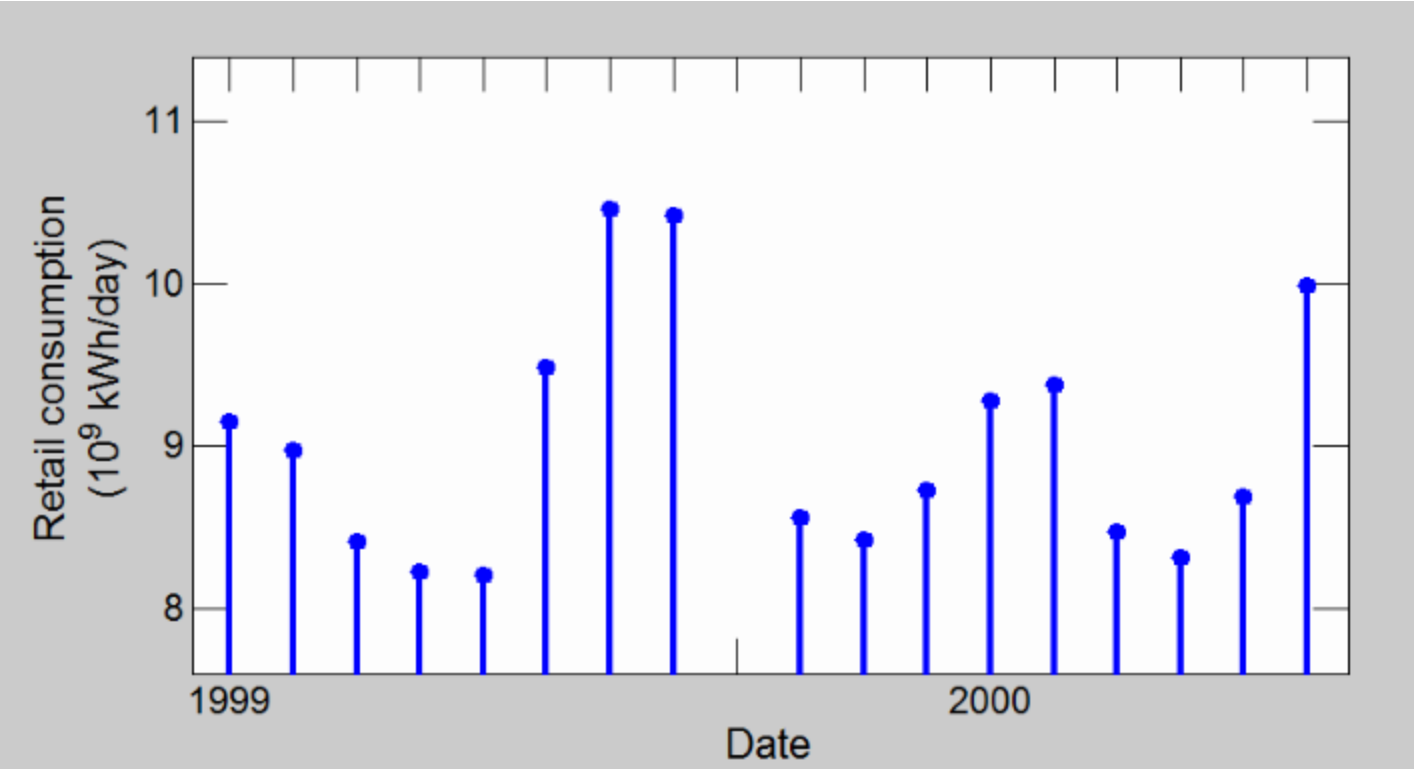
Curve fitting



Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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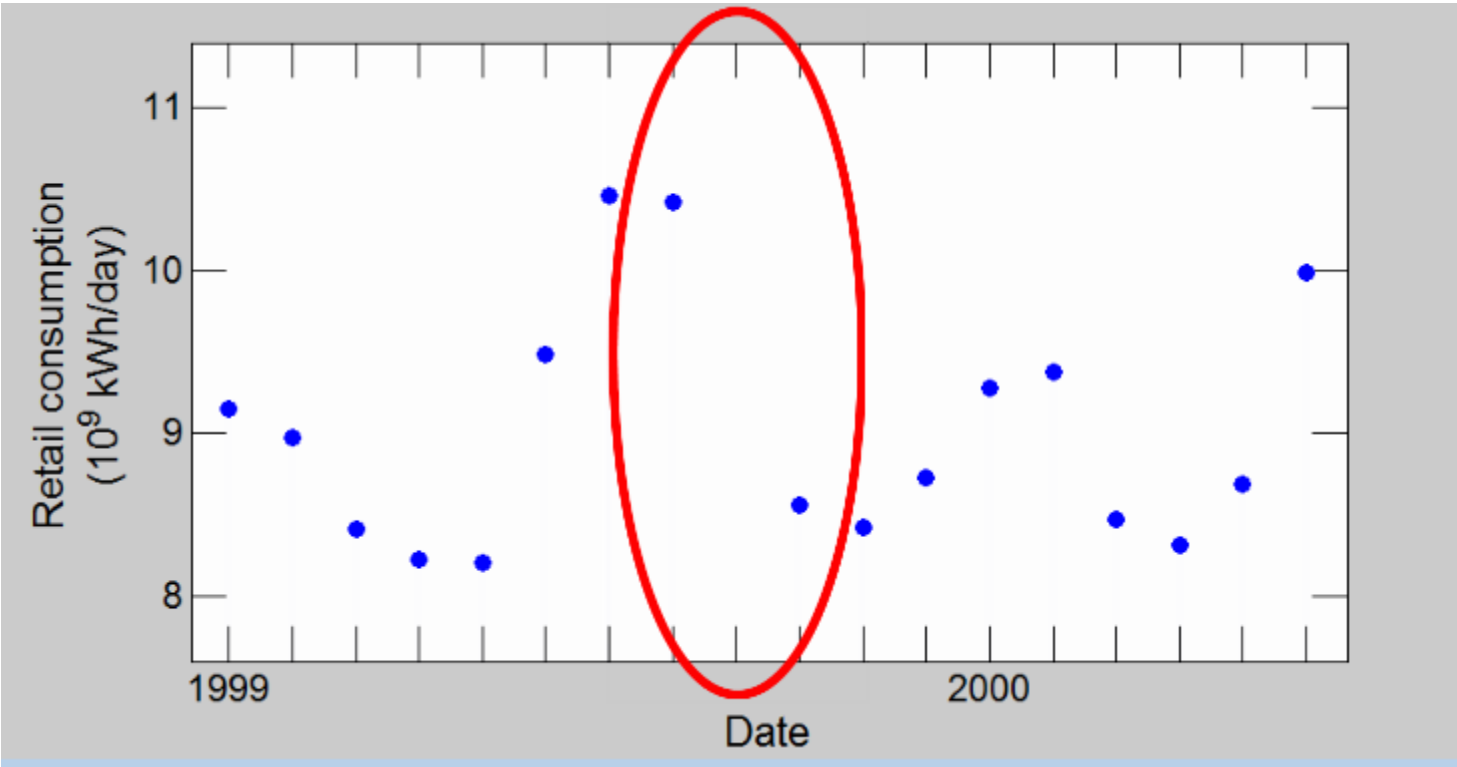
Curve fitting



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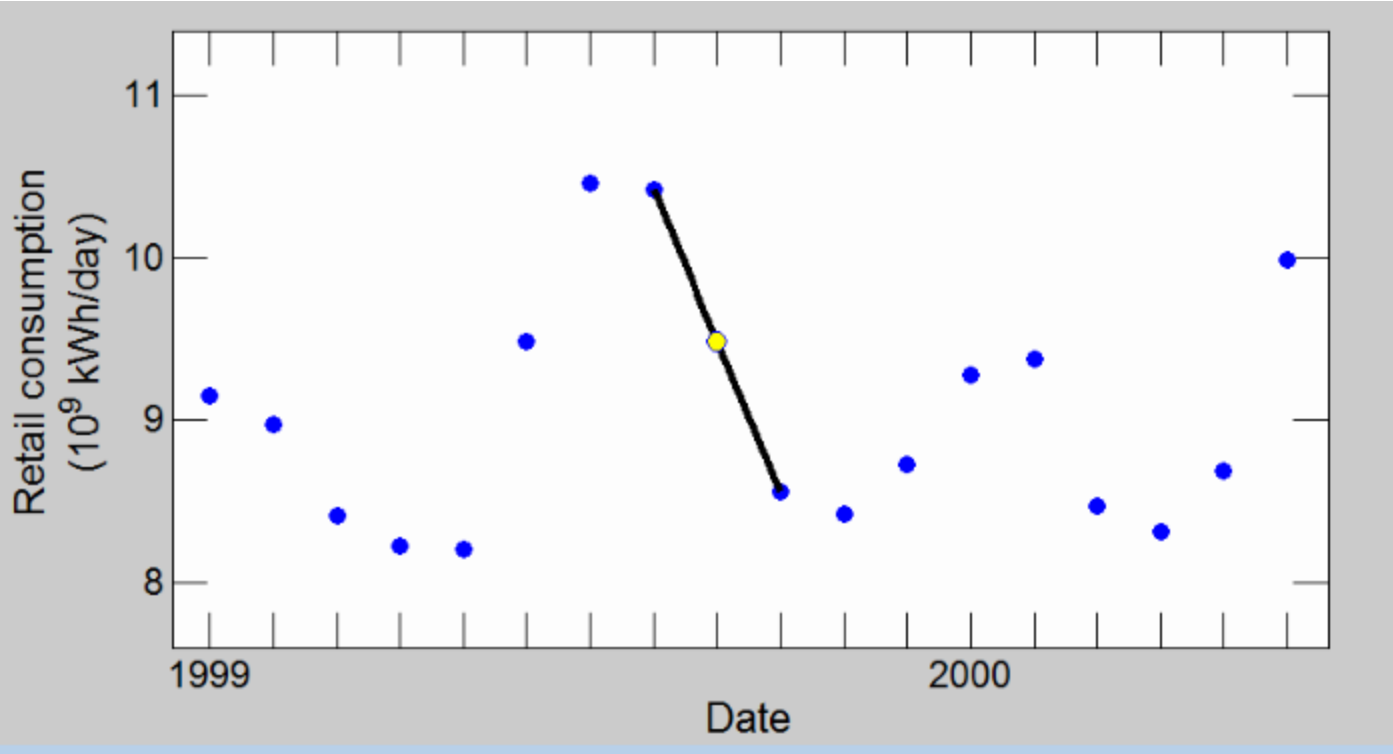
Curve fitting: linear interpolation between 2 points



Source:
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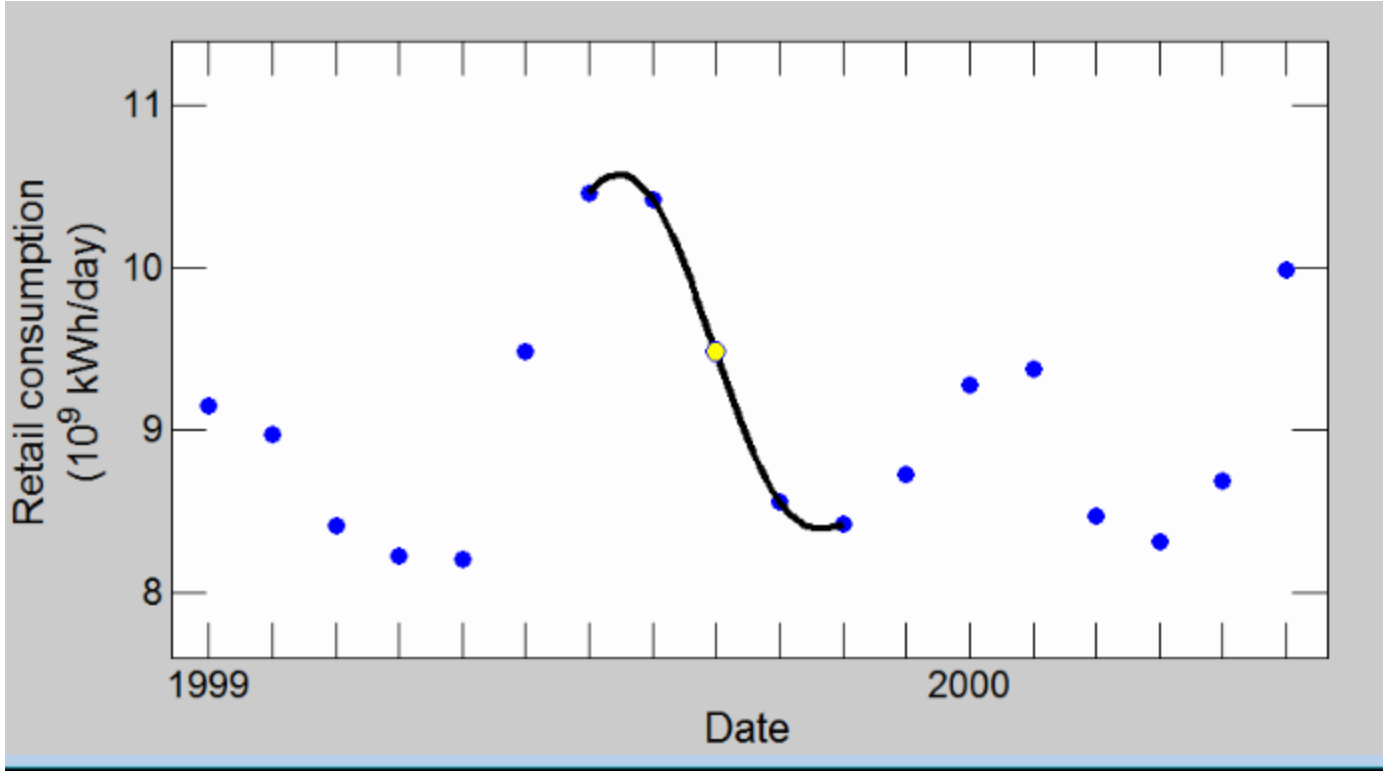
Curve fitting: curve through 4 points



Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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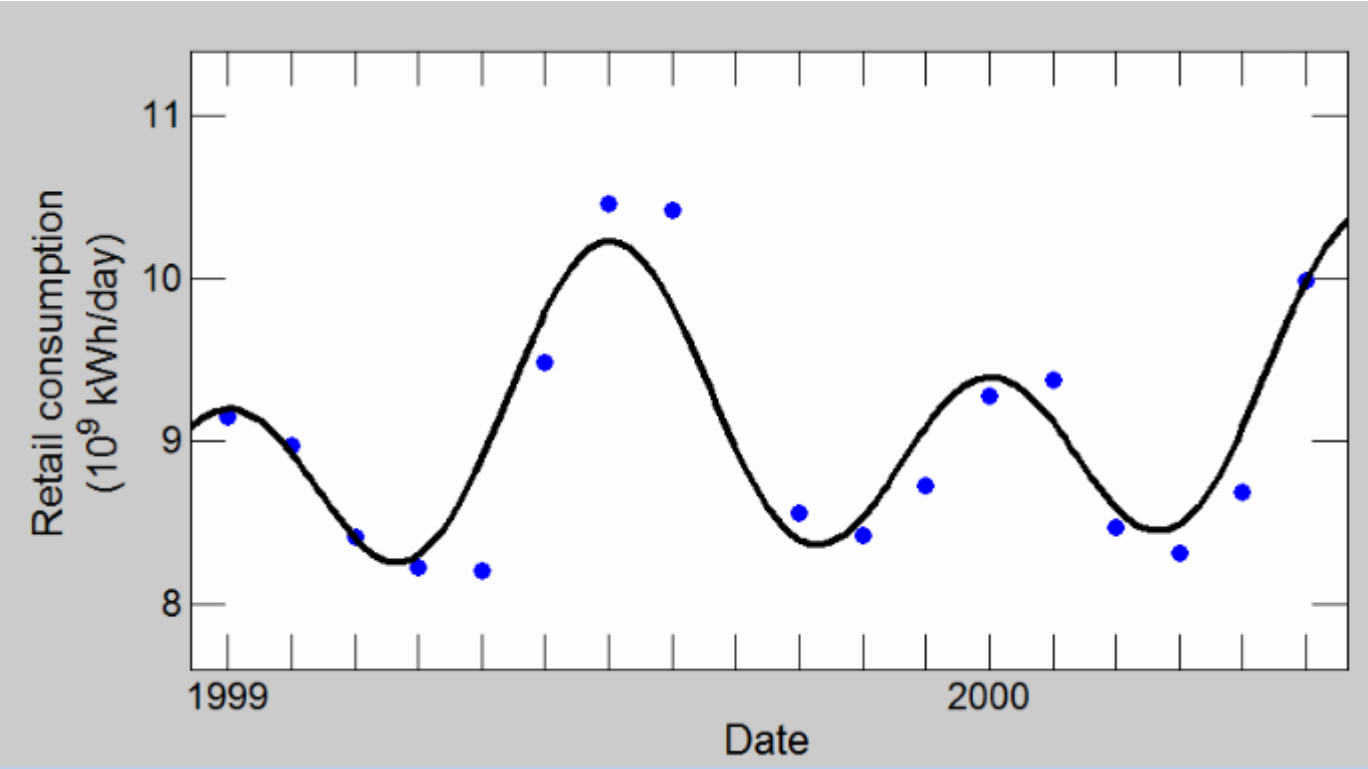
Curve fitting: using all data points



Source:
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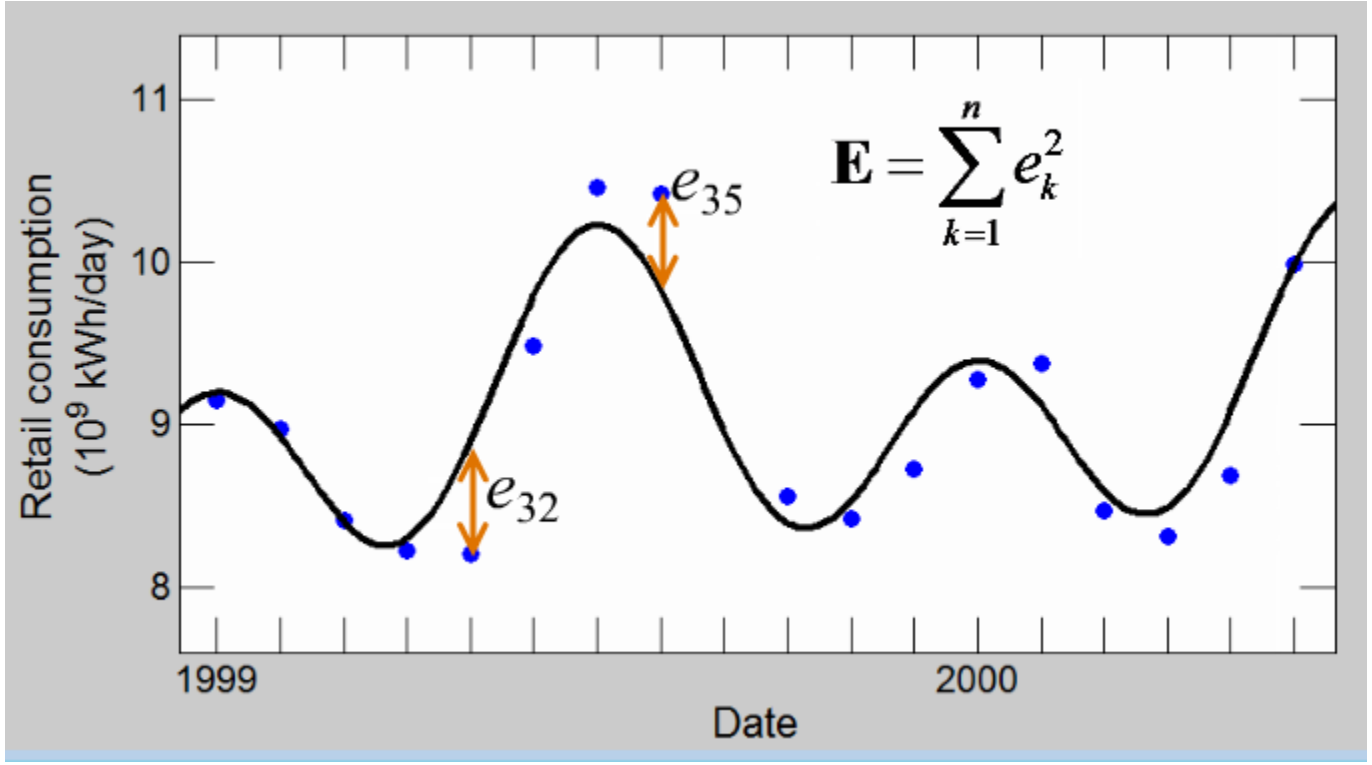
Curve fitting:
using all data points
minimizing error (or least square error) between fit and data



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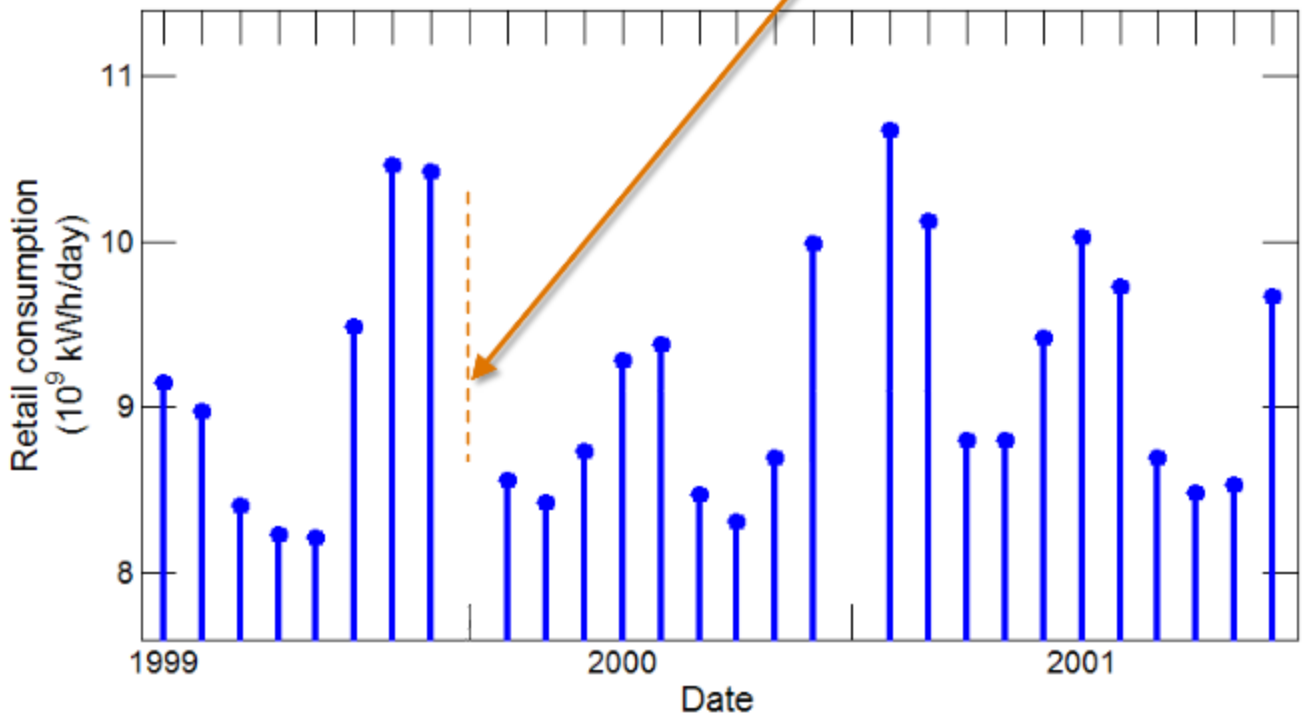
Curve fitting:

Interpolating with MATLAB

```
yi = interp1(x,y,xi);
```



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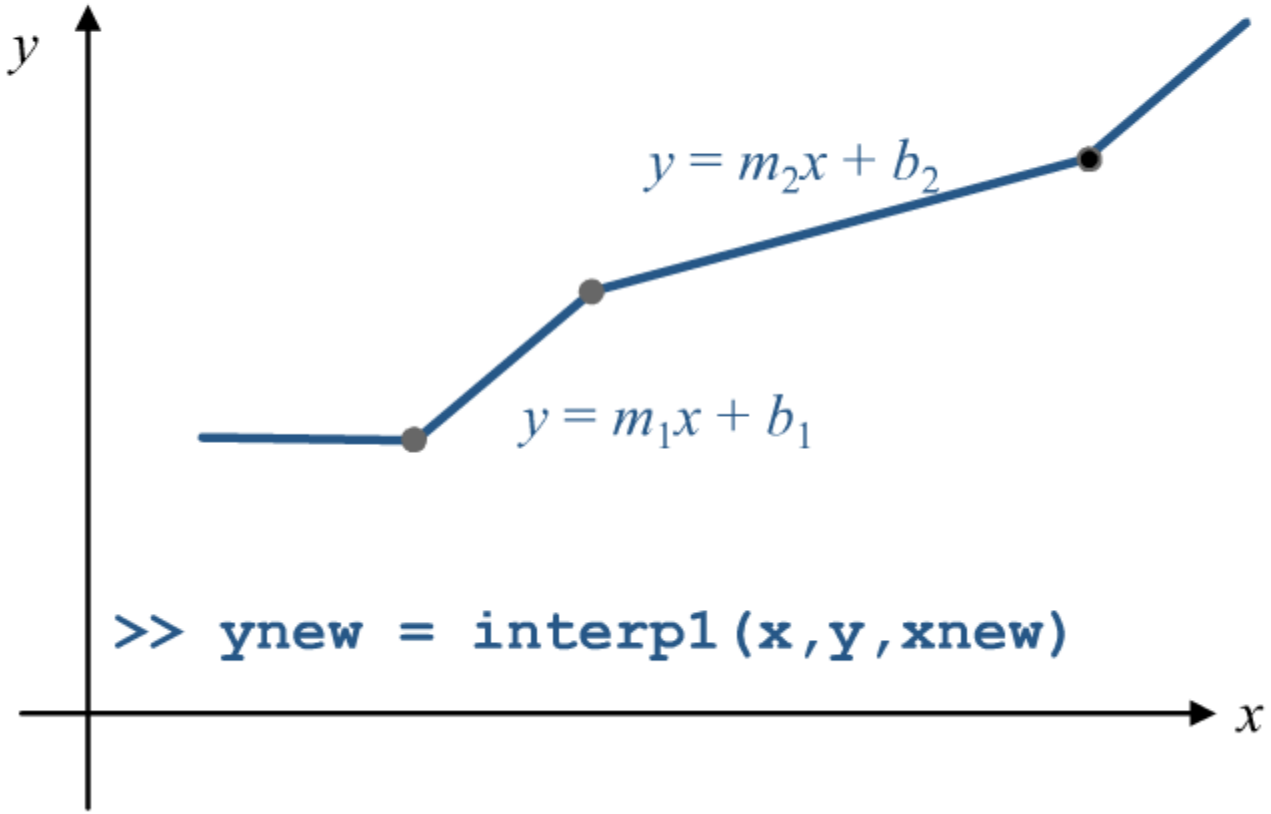
Curve fitting:

Using Piecewise Linear Interpolation on Multiple Data Points




```
>> ynew = interp1(x,y,xnew)
```

Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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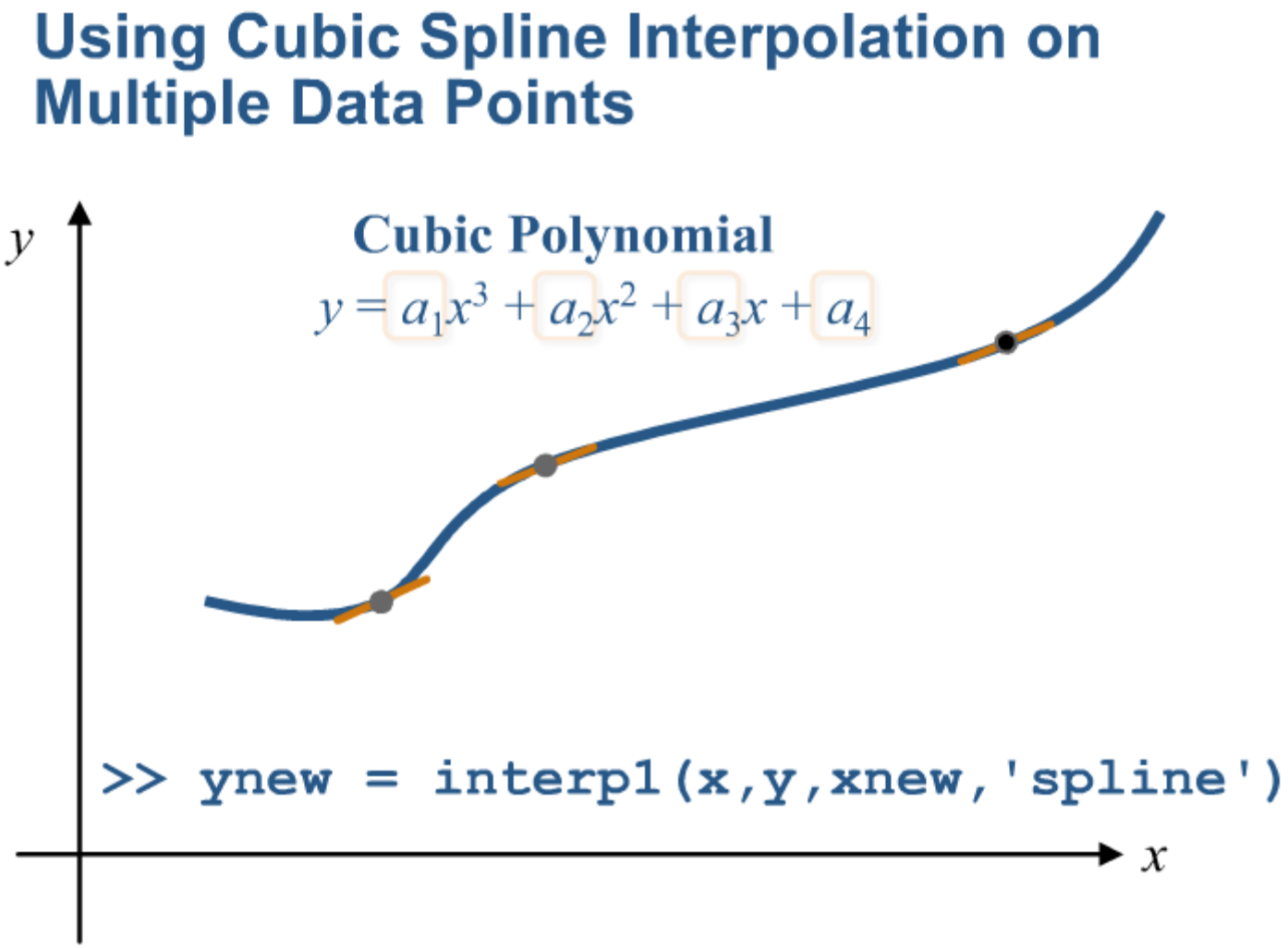
Curve fitting:



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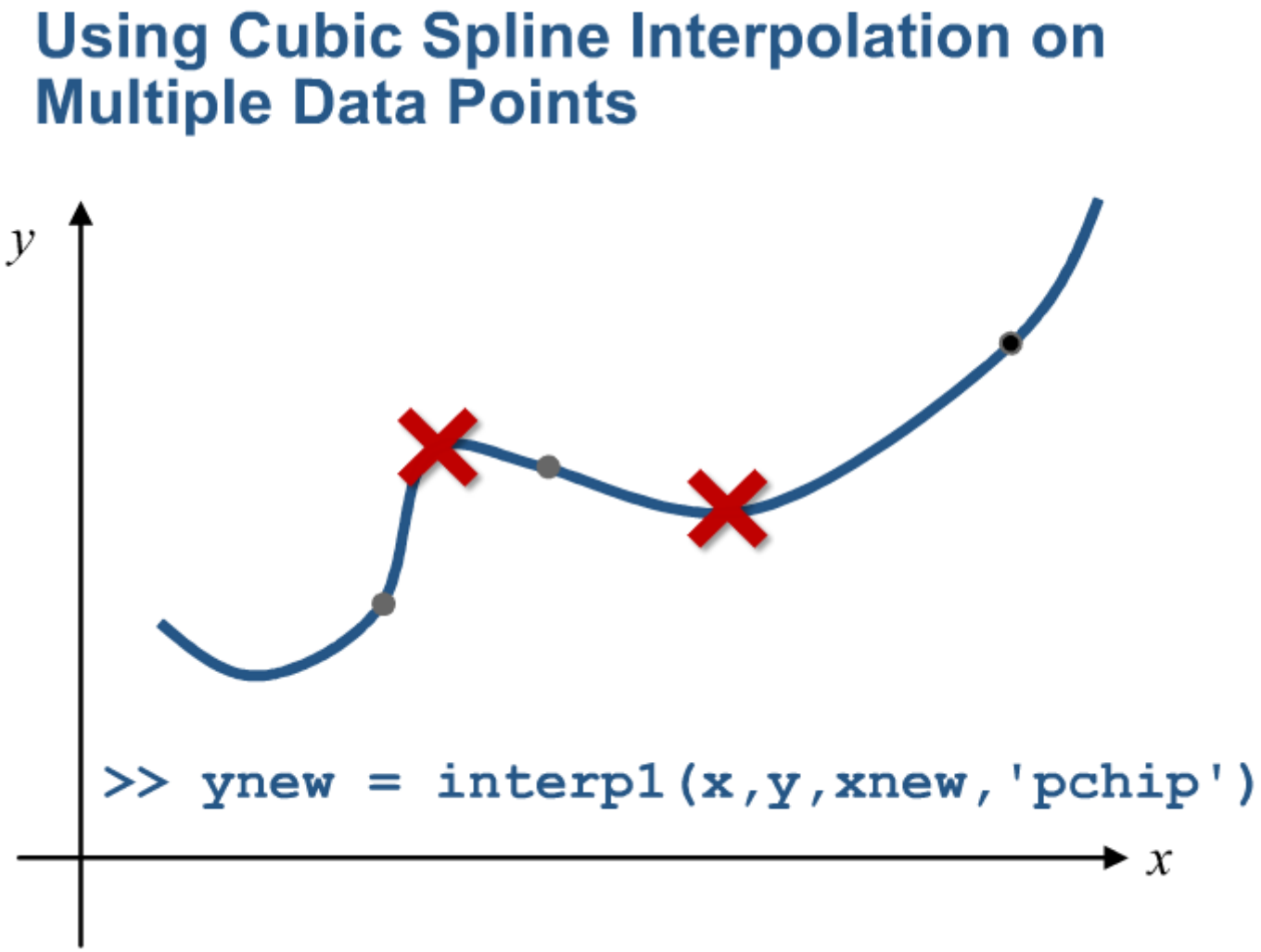
Curve fitting: higher order not necesserily better!



Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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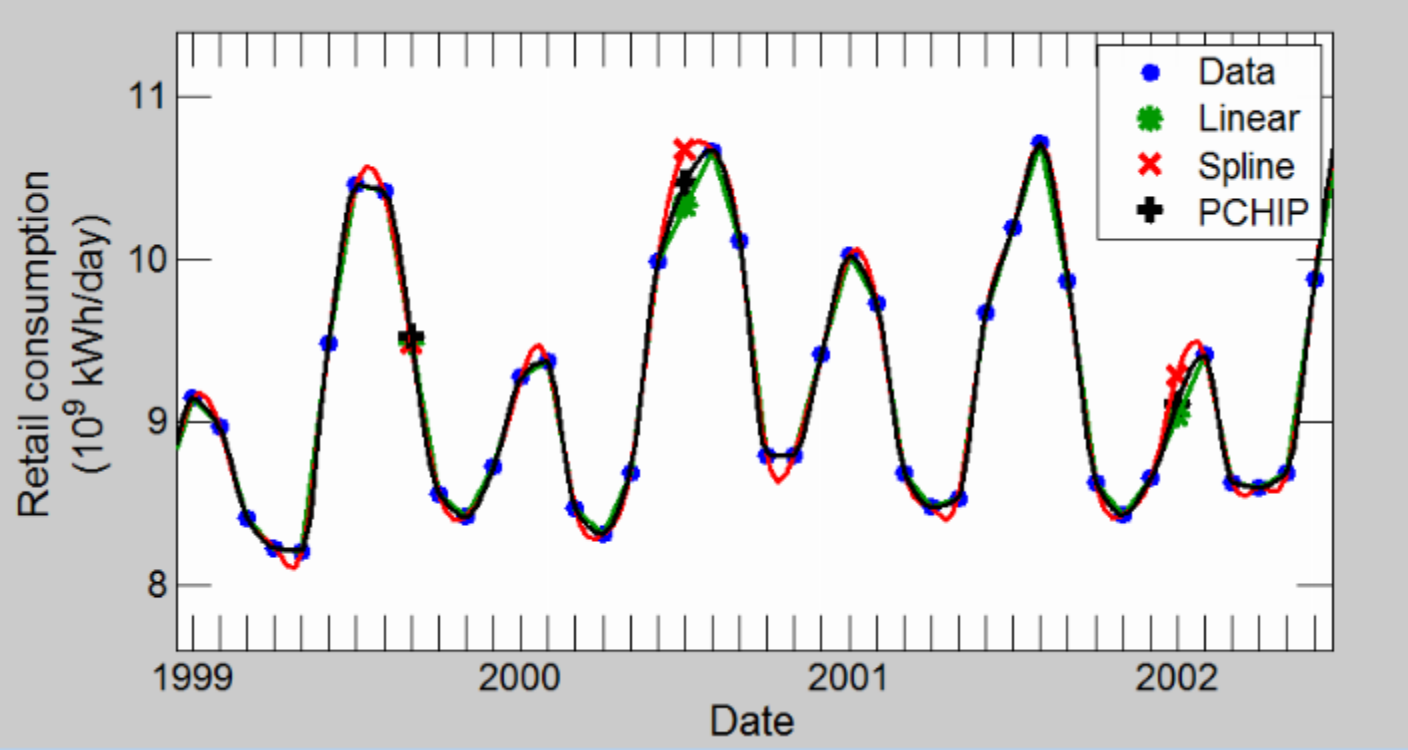
Curve fitting: higher order not necesserily better!



Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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Curve fitting:

Matlab toolboxes:
cftool

Source:
http://www.mathworks.com/academia/student_center/tutorials/computational-math/data-fitting/player.html

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Curve fitting:

To illustrate linear regression, cons ... easurements:

Time (s)	Temperature (deg F)
0	0
1	20
2	60
3	68
4	77
5	110

For the plotted data

```
>> MSE = mean((yhat-y).^2)
MSE =
    95.5000
>> RMSE = sqrt(MSE)
RMSE =
     9.7724
```

```
x = 0:5;
y = [0 20 60 68 77 110];
yhat = 20*x
err = yhat-y
MSE = mean(err.^2)
RMSE = sqrt(MSE)
plot( x,y,'o', x,yhat),title('Linear Estimate'),...
  xlabel('time, s'),ylabel('Temperature, degrees F'),...
  grid,axis([-1,6,-2,120]), legend('measured', 'estimated',4)
```

A measure of the quality of the fit of the linear estimate to the data is the mean squared error (MSE) or the root mean squared error (RMSE)

$$\mathrm{MSE} = \frac{1}{N}\sum_{k=1}^{N}(\hat{y}_k - y_k)^2$$

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}}$$

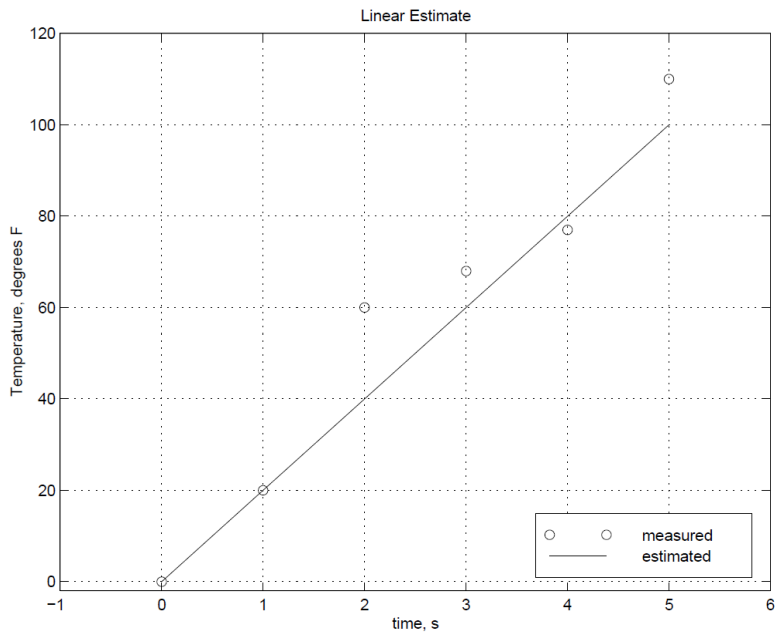

Figure 10.1: A linear estimate

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 20

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Curve fitting:

The best fit is the line $\hat{y} = a_1 x + a_2$ having coefficients a_1 and a_2 that produce the smallest mean squared error (MSE). MSE is expressed as

$$\begin{aligned} \text{MSE} &= \frac{1}{N}\sum_{k=1}^{N}(\hat{y}_k - y_k)^2 \\ &= \frac{1}{N}\sum_{k=1}^{N}(a_1 x_k + a_2 - y_k)^2 \end{aligned}$$

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 2015.

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Curve fitting:

The MSE is minimum when its partial derivatives with respect to each of the coefficients are zero.

$$
\begin{aligned}
\frac{\partial \mathrm{MSE}}{\partial a_1} &= \frac{1}{N}\sum_{k=1}^{N} 2(a_1 x_k + a_2 - y_k)x_k \\
&= \frac{2}{N}\sum_{k=1}^{N} a_1 x_k^2 + a_2 x_k - x_k y_k \\
&= \frac{2}{N}\left[\left(\sum_{k=1}^{N} x_k^2\right) a_1 + \left(\sum_{k=1}^{N} x_k\right) a_2 - \left(\sum_{k=1}^{N} x_k y_k\right)\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathrm{MSE}}{\partial a_2} &= \frac{1}{N}\sum_{k=1}^{N} 2(a_1 x_k + a_2 - y_k) \\
&= \frac{2}{N}\left[\left(\sum_{k=1}^{N} x_k\right) a_1 + N a_2 - \left(\sum_{k=1}^{N} y_k\right)\right] \\
&= 0
\end{aligned}
$$

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 2015.

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Curve fitting:

$$\begin{bmatrix} \sum_{k=1}^{N} x_k^2 & \sum_{k=1}^{N} x_k \\ \sum_{k=1}^{N} x_k & N \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N} x_k y_k \\ \sum_{k=1}^{N} y_k \end{bmatrix}$$

This is a pair of linear equations in two unknowns that can be solved using the methods discussed in the previous section of these notes. The values of a_1 and a_2 determined in this way represent the straight line with the minimum mean squared error.

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 2015.

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Curve fitting:

The MATLAB command to compute the best linear fit to a set of data is `polyfit(x,y,1)`, which returns a coefficient vector of the straight line fitting the data. For the data above:

```
x = 0:5;
y = [0 20 60 68 77 110];
a = polyfit(x,y,1)
yhat = polyval(a,x);
err = yhat - y
MSE = mean(err.^2)
RMSE = sqrt(MSE)
plot(x,y,'o',x,yhat),title('Linear Regression'),...
  xlabel('time, s'),ylabel('Temperature, degrees F'),...
  grid,axis([-1,6,-2,120]),legend('measured','estimated',4)
```

Running this script:

```
a =
   20.8286    3.7619
err =
    3.7619    4.5905  -14.5810   -1.7524   10.0762   -2.0952
MSE =
   59.4698
RMSE =
    7.7117
```

Thus, the best linear fit is

$$\hat{y} = 20.8286x + 3.7619$$

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 2015.

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Curve fitting

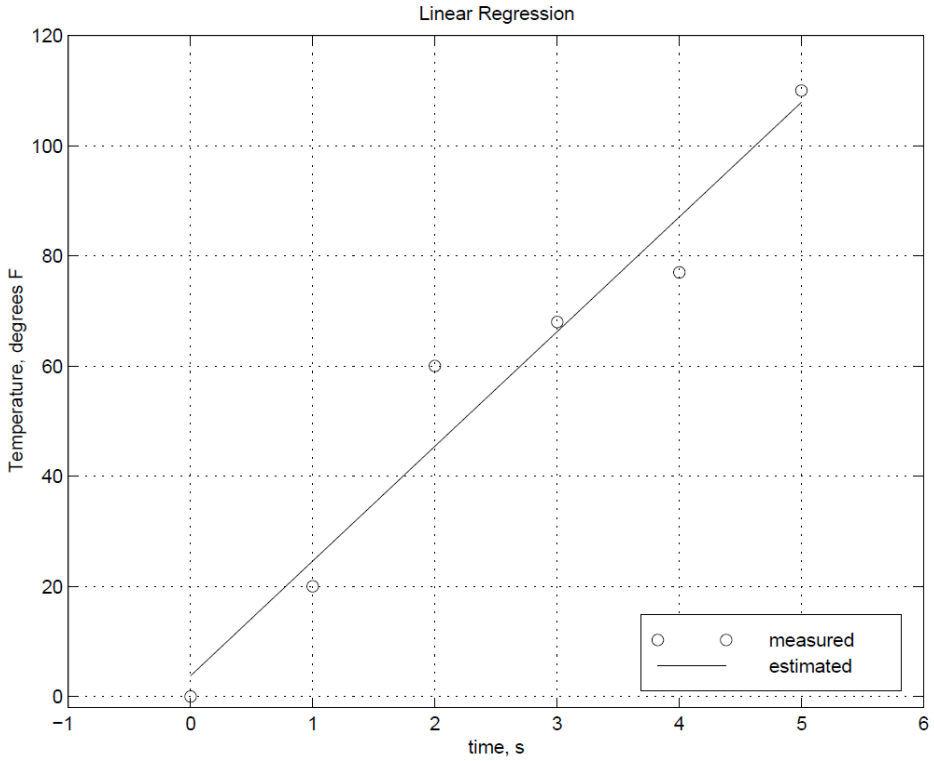

Figure 10.2: Linear regression curve fit

```
>> yi = polyval(a,2.5)
yi =
   55.8333
```

Source:
Chapman, Stephen J. *MATLAB programming for engineers*. Nelson Education, 2015.