
$$v = \dot{s} = ds/dt;\ a = \dot{v} = dv/dt = \ddot{s} = d^2s/dt^2.\ v_t = \dot{s} = \rho\dot{\beta};\ \ v_n = 0;\ a_t = \dot{v} = \rho\ddot{\beta} + \dot{\rho}\dot{\beta};\ a_n = v^2/\rho = \rho\dot{\beta}^2.$$

$$v_r = \dot{r};\ v_\theta = r\dot{\theta};\ v_z = \dot{z};\ a_r = \ddot{r} - r\dot{\theta}^2;\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta};\ a_z = \ddot{z};\ v_R = \dot{R};\ v_\theta = R\dot{\theta}\cos\phi;\ v_\phi = R\dot{\phi};\ \omega = \dot{\theta};\ \alpha = \dot{\omega};\ \omega d\omega = \alpha d\theta$$

$$\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r};\ \mathbf{a}_n = \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r});\ \mathbf{a}_t = \boldsymbol{\alpha}\times\mathbf{r};\ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A|B} = \mathbf{v}_B + (\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{v}_{\text{rel}};\ \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A|B} = \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}};$$

$$\mathbf{G} = \sum m_i\mathbf{v}_i\ ;\sum\mathbf{F} = \dot{\mathbf{G}};\ \mathbf{H}_O = \sum\mathbf{r}_i\times m_i\mathbf{v}_i\ ;\sum\mathbf{M}_O = \dot{\mathbf{H}}_O\ ;\mathbf{H}_G = \sum\boldsymbol{\rho}_i\times m_i\dot{\boldsymbol{\rho}}_i\ ;\sum\mathbf{M}_G = \dot{\mathbf{H}}_G\ ;$$

$$\sum M_O = I_O\alpha;\ \sum M_P = \bar{I}\alpha + m\bar{a}d;\ \text{Lei senos: } L_A/\operatorname{sen}\theta_A = L_B/\operatorname{sen}\theta_B\ ;\ \text{Lei cosenos: } L_A^2 = L_B^2 + L_C^2 - 2L_BL_C\cos\theta_A$$

$$T_1 + U_{1-2} = T_2\ ;\ T_1 + V_1 + U'_{1-2} = T_2 + V_2\ ;\ T = \sum\frac{1}{2}m_i\|\mathbf{v}_i\|^2\ ;\ U_{1-2} = \int_1^2\mathbf{v}\cdot d\mathbf{s}\ ;\ V = \frac{1}{2}kx^2\ ;\ V = mgh$$

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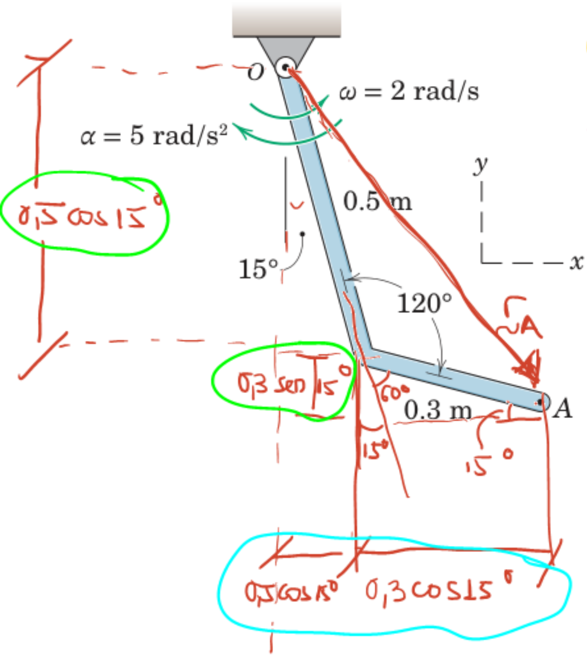
1 [25] A barra pendular da figura gira em torno de O. Determine: (a) [10] a velocidade; (b) [15] aceleração – do ponto A nas condições mostradas na figura.



$\underset{\sim}{\omega} = 2\,\hat{\underset{\sim}{k}}$

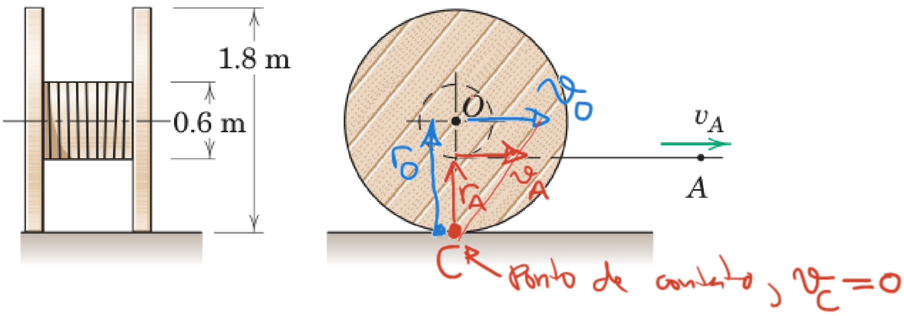
$\underset{\sim}{\alpha} = -5\,\hat{\underset{\sim}{k}}$

$\underset{\sim}{r}_A = (0{,}3\cos 15^\circ + 0{,}5\cos 15^\circ)\,\hat{\underset{\sim}{\imath}}$
$- (0{,}3\,\text{sen}\,15^\circ + 0{,}5\cos 15^\circ)\,\hat{\underset{\sim}{\jmath}}$

(a) $\underset{\sim}{v}_A = \underset{\sim}{\omega} \times \underset{\sim}{r}_A = 1{,}12\,\hat{\underset{\sim}{\imath}} + 0{,}84\,\hat{\underset{\sim}{\jmath}}$

(b) $\underset{\sim}{a}_A = \underset{\sim}{a}_n + \underset{\sim}{a}_t = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_A)$
$+ \underset{\sim}{\alpha} \times \underset{\sim}{r}_A = -4{,}48\,\hat{\underset{\sim}{\imath}} + 0{,}147\,\hat{\underset{\sim}{\jmath}}$

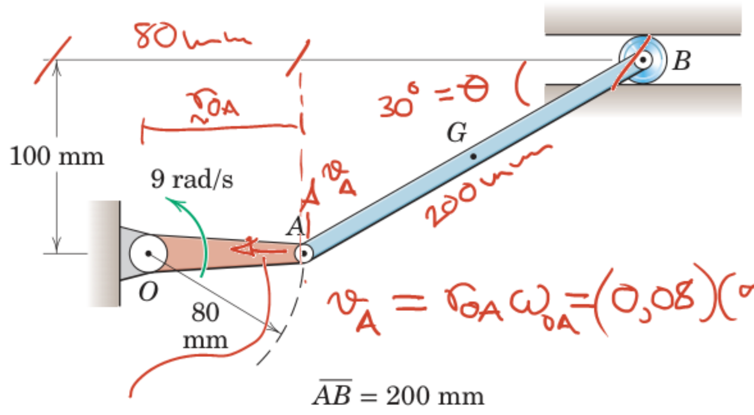
2 [25] O carretel da figura rola sem deslizar puxado pelo cabo em A com $v_A = 0{,}8$ m/s. Calcule: (a) [10] a velocidade do centro do carretel e (b) [15] sua velocidade angular. Dica: *NÃO cometa o ERRO de achar que o carretel se deslocará para a esquerda*!!!!!



(a) $v_0 = \omega r_0$, $v_A = \omega r_A$ $\Rightarrow$ $\dfrac{v_0}{v_A} = \dfrac{r_0}{r_A}$ $\Rightarrow$ $v_0 = \left(\dfrac{r_0}{r_A}\right) v_A = \left(\dfrac{0{,}9}{0{,}6}\right)(0{,}8) = 1{,}2 \text{ m/s}$ //

(b) $\omega = \dfrac{v_0}{r_0} = \dfrac{1{,}2}{0{,}9} = 1{,}33 \text{ rad/s}$ //

3 [25] A barra OA gira no sentido mostrado com velocidade angular de 9 rad/s. Determine a aceleração angular da barra AB,

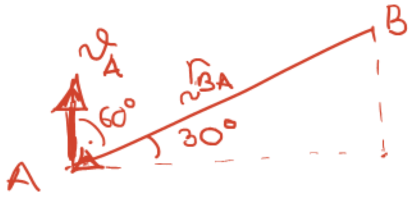


$\theta = \text{arcsen}\left(\frac{100}{200}\right) = \text{arcsen}\left(\frac{1}{2}\right) = 30°$

$v_A = r_{OA}\,\omega_{OA} = (0{,}08)(9) = 0{,}72\ \text{m/s}.$

aceleração normal em A:

$a_A = \omega^2 r_{OA} = (9^2)(0{,}08) = 6{,}48\ \text{m/s}^2$



$\omega_{AB} \times r_{BA} = v_A$

$r_{AB}\,\omega_{AB}\,\text{sen}\,60° = v_A$

$(0{,}2)\left(\frac{\sqrt{3}}{2}\right)\omega_{AB} = 0{,}72$

$\omega_{AB} = 4{,}16\ \text{rad/s}$ (horário)

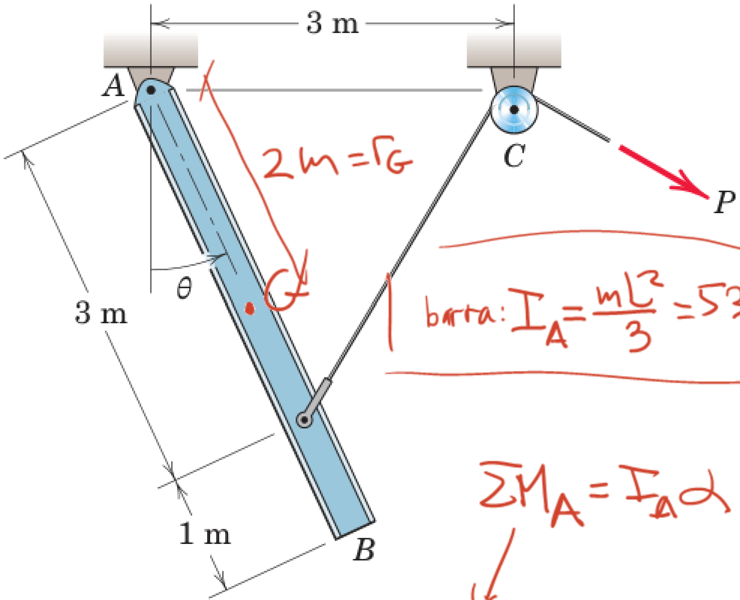
$$a_B = a_A + a_{B/A} = a_A + \alpha_{AB} \times r_{B/A} - \omega_{AB} \times (\omega_{AB} \times r_{B/A})$$

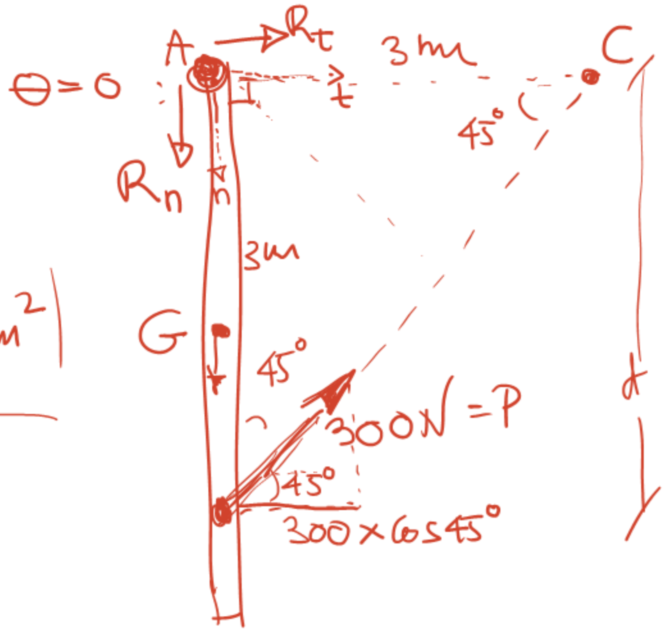
$$a_B\,\hat{i} = -6{,}48\,\hat{i} + \alpha_{AB}\,\hat{k} \times 0{,}2(\cos 30°\,\hat{i} + \text{sen}\,30°\,\hat{j}) - 4{,}16\,\hat{k} \times \left[4{,}16\,\hat{k} \times 0{,}2(\cos 30°\,\hat{i} + \text{sen}\,30°\,\hat{j})\right]$$

2 incógnitas a_B e α_{AB}; igualando os coef. de $\hat{i}$ e $\hat{j}$

$a_B = -10{,}5\ \text{m/s}^2$, $\alpha_{AB} = 10\ \text{rad/s}^2$

4 [25] A barra articulada no ponto A da figura tem 100 kg . Supondo que o ângulo θ é igual a ZERO no exato instante em que a força $P = 300$ N é aplicada. (a) [15] Determine para esse instante a aceleração angular da barra que fará θ crescer. (b) [10] Determine também a força de reação F_A no pino A. (Obs: o momento de inércia da barra em relação a A possui fórmula $I_A = \frac{1}{3}mL^2$.)





barra: $I_A = \frac{mL^2}{3} = 533\ kgm^2$

$\sum M_A = I_A \alpha$

P é a única força que causa momento

$\underbrace{(300)\left(\frac{\sqrt{2}}{2}\right)}_{P\cos 45}\underbrace{(3)}_{\text{(braço de alavanca d=3m)}} = 533\,\alpha \Rightarrow \boxed{\alpha = 1{,}19\ rad/s^2}$ [a]

(b)

para o C.G. da barra

$\sum F_n = m a_n^{\,0}$: $\underbrace{R_n}_{\text{reações}} + \underbrace{300\cos 45}_{\text{tensão P}} - \underbrace{(100)(9{,}81)}_{\text{Peso}} = 0 \Rightarrow R_n = 769\,N$

$\sum F_t = m a_t$: $R_t + 300\,sen45 = (100)\underbrace{(2)(1{,}19)}_{\text{acel. do C.G.}} \Rightarrow R_t = 26{,}5$

$a_t = r_G\alpha$ (r_G = 2m, α = 1,19 rad/s²)